JBF-1603010102010300 Seat No. _____

M. Sc. (Sem. I) Examination

December – 2019

Physics : Paper - CT-03

(Quantum Mechanics-I) (New Course)

Time : **2:30** Hours] [Total Marks : **70**

Instructions : (1) Attempt all questions.
(2) All questions carry equal marks.
(3) Marks assigned are indicated in brackets on RHS.
(4) Mathematical symbols have usual meanings.

1 Answer in brief any seven : **14**

(a) Prove that $[H, a] = -\hbar w a$. **2**

(b) Define both the dimensionless coordinates ξ and $\in$. **2**

(c) How a^t acts as a creation operator ? **2**

(d) Prove that $\left[J_+, J_-\right] = 2\hbar J_2$. **2**

(e) Depict the classical and quantum probability curves for n = 0 and n = 1. **2**

(f) In the case of WKB approximation one obtain $|\psi|^2 \propto 1/p$, where p is a momentum of a particles, how this leads to semi-classical approximation ? **2**

(g) What is trial wave function ? How it is selected ? **2**

(h) In the time dependent perturbation draw the following highly peaked function. **2**

$$\sin^2\left[(w_{mi} \pm w)\, t/2\right] \Big/ \left[(w_{mi} \pm w)/2\right]^2$$

and prove the time-energy uncertainty for the peak value.

(i) In the time independent perturbation theory the first order equation is given as **2**

$$\left(E_k - E_m\right)C_k^{(1)} + H'_{km} - W^{(1)}\,\delta_{km} = 0$$

then find out $W^{(1)}$ in the case of k = m.

(j) Using which method the schrodinger equation in polar coordinates is separated into radial and angular equations ? Write the wave function ψ having angular and radial parts. **2**

2 Answer any two :

(a) In obtaining the solution of one dimensional harmonic oscillator, derive up to the following equation only **7**

$$\frac{d^2h}{d\xi^2} - 2\xi\frac{dh}{d\xi} + h\left(\in -1\right) = 0$$

(b) Explain the harmonic oscillator energy spectrum. **7**

(c) By defining the operators "a" and "a^+", derive the oscillator Hamiltonian in terms of "a" and "a^+" as : **7**

$$H = \hbar w\left(a^+a + \frac{1}{2}\right)$$

3 (a) Obtain the gradient operator $\vec{\nabla}$ in spherical polar coordinates as : **7**

$$\vec{\nabla} = \vec{e}_r\frac{r}{\partial r} + \vec{e}_\theta\,\frac{1}{r}\frac{\partial}{\partial\theta} + \vec{e}_\phi\,\frac{1}{r\sin\theta}\frac{\partial}{r\phi}$$

(b) Using the concept of operators for angular momentum, obtain the following relations : **7**

(i) $\lambda_J - m_{\max}\left(m_{\max}+1\right)\hbar^2 = 0$

(ii) $\lambda_J - m_{\min}\left(m_{\min}-1\right)\hbar^2 = 0$

OR

3 (a) Solve the harmonic oscillator in polar coordinates and obtain the energy eigen values **7**

$$E_{n,l} = \left(2n + l + \frac{3}{2}\right)\hbar w$$

(b) In the time independent perturbation theory consider the non-degenerate case and obtain the first order equation as follows : **7**

$$\left[E_k - E_m\right]C_k^{(1)} + H'_{km} - W^{(1)}\delta_{km} = 0$$

4 Answer any two :

(a) Obtain the energy values of harmonic oscillator by WKB approximation as $E = \left(n + 1/2\right)\hbar w$. **7**

(b) In the time dependent perturbation theory derive for the first order equation as **7**

$$C_m(t) = \delta_{mi} - \frac{i}{\hbar}\int_0^t dt' e^{i\left(E_m - E_i\right)t/\hbar}\left\langle \phi_m \mid H,(t') \mid \phi_i \right\rangle \left|C_m(t)\right|^2$$

indicates what ?

(c) Discuss the variation method. **7**

5 Write any two notes on following :

(a) WKB approximation. **7**

(b) Fermi golden rule. **7**

(c) Spherical harmonics. **7**

(d) The raising, lowering and number operators. **7**
